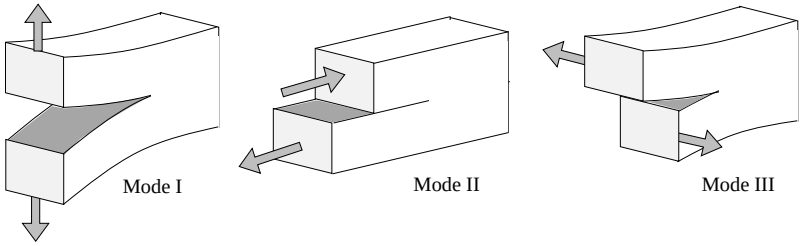
Fracture Mechanics Reference Sheet

Created by Dr. Gary S. Prinz for CVEG 563V

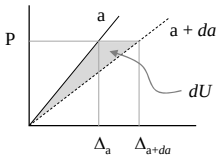
Fracture Modes:



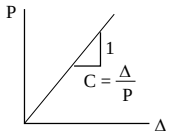
Fracture Toughness Parameters:

$\mathcal{G}$ → Energy Release Rate → Net change in potential energy per incremental crack growth $\left(\frac{dU}{da}\right)$ → Global Behavior → $\mathcal{G} = \frac{\pi\sigma^2 a}{E}$

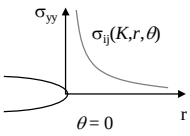
a.k.a. driving force



$\mathcal{G} = \frac{P^2}{2B}\left(\frac{\partial C}{\partial a}\right)$

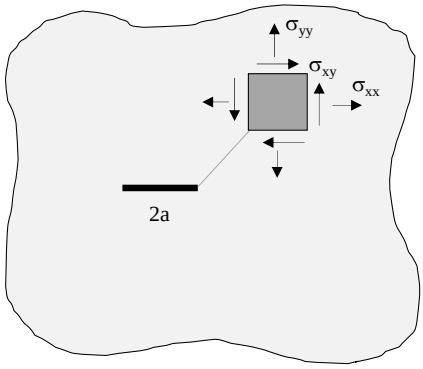


K → Stress Intensity Factor → Characterizes stresses, strains (displacements) at the crack tip → Local Parameter → $K = Y\sigma\sqrt{\pi a}$



Similitude:

2-D Planar Crack-Tip Stresses



$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$

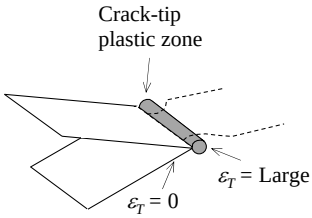
σ_{ij} is characterized by K_I

$\therefore$ equal K_I ensures similitude given the same material properties.

K_{IC} (Small test) $=$ K_{IC} (Structural Application)

Critical K_I value

3-D Out-of-Plane Crack-Tip Stresses



$$\varepsilon_T = \frac{\nu\sigma}{E} = \nu\varepsilon_L$$

For Metals:

$\varepsilon_T \approx \frac{1}{3}\varepsilon_L$ Elastic Zone

$\varepsilon_T \approx \frac{1}{2}\varepsilon_L$ Plastic Zone

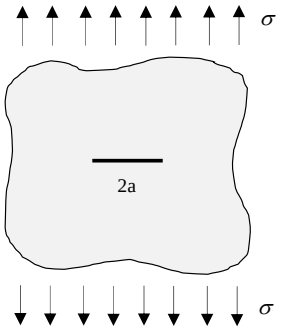
Plane Strain	Plane Stress
$\varepsilon_z = 0$	$\sigma_z = 0$
$\sigma_z \neq 0$	$\varepsilon_z \neq 0$

Considering both in-plane and out-of-plane crack tip stresses, similitude requires:

- Same material
- Equal K (meaning same in-plane crack tip stresses)
- Equal state of stress (meaning same out-of-plane crack tip stresses)

Common Flaw Configurations:

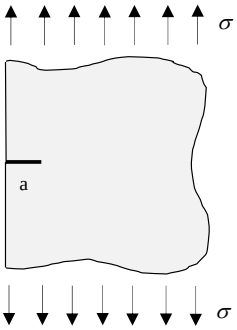
Center through-thickness crack in infinite plate



$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 1$

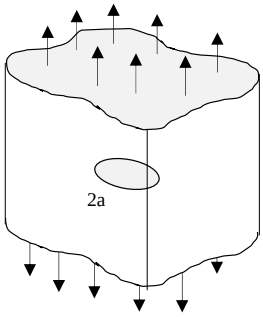
Edge through-thickness crack in infinite plate



$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 1.12$

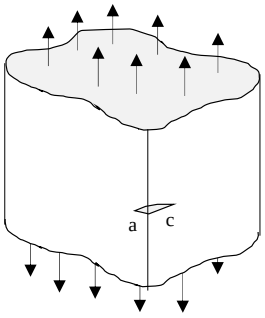
Circular void flaw (penny shape) in infinite plate



$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 2/\pi$

Quarter-elliptical corner flaw in infinite plate

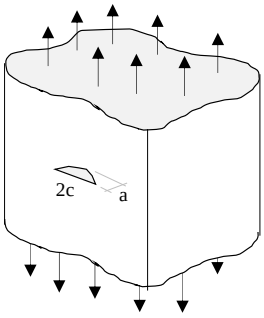


$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 1.21/\Phi$

$$\Phi = \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a^2}{c^2}\right)$$

Semi-elliptical edge flaw in infinite plate

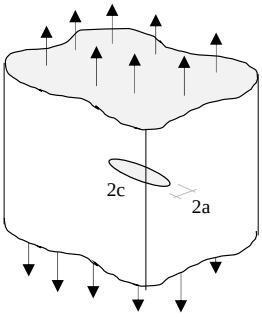


$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 1.12/\Phi$

$$\Phi = \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a^2}{c^2}\right)$$

Elliptical void flaw in infinite plate



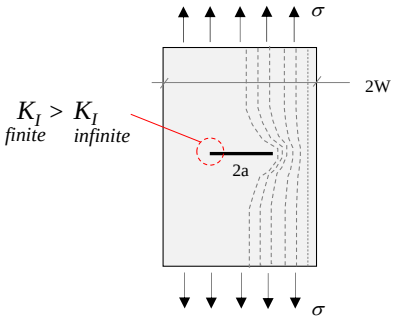
$$K_I = Y\sigma\sqrt{\pi a}$$

$Y = 1/\Phi$

$$\Phi = \frac{3\pi}{8} + \frac{\pi}{8}\left(\frac{a^2}{c^2}\right)$$

Corrections for Finite Size:

Center through-thickness crack in finite plate



$$K_I = Y\sigma\sqrt{\pi a}$$

$$Y_f = \left[\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)\right]^{1/2}$$

(Derived from infinite plate with multiple cracks)

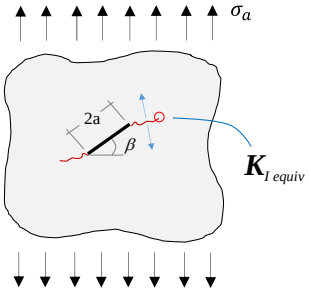
or

$$Y_f = \sec^{1/2}\left(\frac{\pi a}{2W}\right)$$

(Derived from FEA crack tip stresses)

Mixed Mode Fracture (Inclined Crack):

Combined Mode I - Mode II



$$K_{I,\ equiv} = K_I cos^3\left(\frac{\theta_m}{2}\right) - 3K_{II} cos^2\left(\frac{\theta_m}{2}\right) sin\left(\frac{\theta_m}{2}\right)$$

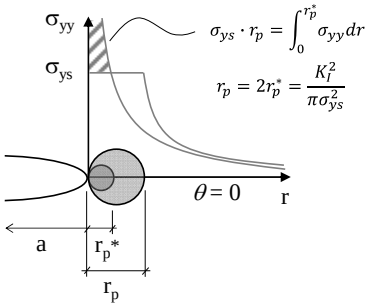
$$\theta_m = 2tan^{-1}\left(\frac{K_I}{4K_{II}} \pm \sqrt{\left(\frac{K_{II}}{4K_{II}}\right)^2 + \frac{1}{2}}\right)$$

$$K_I = \sigma\sqrt{\pi a} = cos^2(\beta) \cdot \sigma_a\sqrt{\pi a}$$

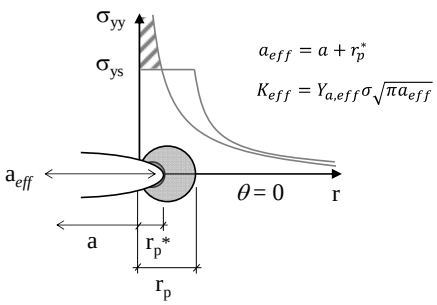
$$K_{II} = \tau\sqrt{\pi a} = \cos(\beta)\sin(\beta) \cdot \sigma_a\sqrt{\pi a}$$

Crack-Tip Plasticity:

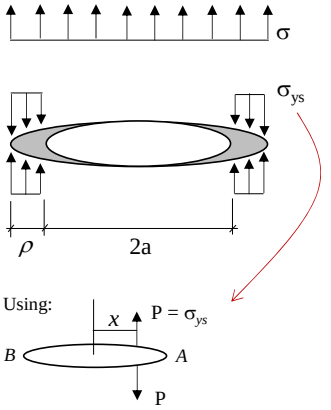
Plastic Re-distribution at Crack Tip



Irwin Plastic Zone Correction



Dugdale Strip Yield Approach



Choosing ρ such that $K = 0$ $\quad [K_{I,\sigma} = -K_{I,\rho}]$

$$K_{I,\rho} = \frac{\sigma_{ys}}{\sqrt{\pi a}} \int_a^{a+\rho} \left(\sqrt{\frac{a+x}{a-x}} + \sqrt{\frac{a-x}{a+x}}\right) dx$$

$$K_{I,\sigma} = \sigma\sqrt{\pi(a+\rho)}$$

$$\rho = \frac{\pi K_I^2}{8\sigma_{ys}^2}$$